On the Cartesian product of arbitrarily partitionable graphs

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A graph $G = (V, E)$ is called arbitrarily partitionable if for every partition $\tau = (\tau_1, \ldots, \tau_k)$ of $|G|$, there exists a partition $(V_1, \ldots, V_k)$ of $V$ such that each $V_i$ induces a connected subgraph of order $n_i$. Clearly, each traceable graph is arbitrarily partitionable.

We deal with the following conjecture.

**Conjecture 1.** If $G$ and $H$ are arbitrarily partitionable graphs, then the Cartesian product $G \square H$ is also arbitrarily partitionable.

This conjecture seems to be difficult, therefore we consider some weaker versions of it. In particular, we assume that one of the factors of the Cartesian product is traceable. We also consider modifications of the notion of arbitrarily partitionable graphs.